Covariant many-fingered time Bohmian interpretation of quantum field theory

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Abstract

The Bohmian interpretation of the many-fingered time (MFT) Tomonaga-Schwinger formulation of quantum field theory (QFT) describes MFT fields, which provides a covariant Bohmian interpretation of QFT without introducing a preferred foliation of spacetime.

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1 Introduction

The Bohmian deterministic interpretation of quantum field theory (QFT) [1, 2, 3, 4, 5] is a promising approach to the general solution of the problem of measurement in QFT. A particular advantage of this interpretation is the fact that it automatically solves the problem of time in canonical quantum gravity [3, 6, 7, 8, 9]. In addition, this interpretation might play an important role in some areas of high-energy physics, such as quantum cosmology [10, 11, 12, 13, 14] and noncommutative theories [15]. An important problem of this interpretation is the fact it requires a preferred foliation of spacetime, which makes it noncovariant [3, 9]. The source of this problem can be traced back to the fact that the functional Schrödinger equation of QFT is also noncovariant because it also requires a preferred foliation of spacetime.

The problem of noncovariance of the Schrödinger equation can be solved by replacing the usual time-dependent Schrödinger equation with the *many-fingered time* (MFT) Tomonaga-Schwinger equation [16, 17], which does not involve a preferred foliation of spacetime. In this formulation, the quantum state is a functional of an arbitrary timelike hypersurface. In a manifestly covariant formulation introduced in [18], the hypersurface does not even need to be timelike.

The covariance of the Tomonaga-Schwinger equation suggests the possibility to formulate a covariant Bohmian interpretation based on the Tomonaga-Schwinger equation, rather than on the Schrödinger equation. Indeed, such an attempt has recently been performed in [19]. (For other recent approaches to the covariant Bohmian formulation of relativistic quantum mechanics and QFT, see also [20, 21]). However, the attempt in [19] was based on a Bohmian equation of motion with a *single* time, so some extra rule that determined the foliation was still necessary. In

contrast with [19], in this paper we observe that a natural Bohmian interpretation of the MFT Tomonaga-Schwinger equation involves a MFT Bohmian equation of motion, which avoids the need for an extra rule that determines the foliation.

The paper is organized as follows. After reviewing the MFT formulation of QFT in Sec. 2, we discuss the MFT Bohmian interpretation in Sec. 3. In Sec. 4 we demonstrate that all results of this paper can be written in a manifestly covariant form, which leads to a covariant form of the Bohmian interpretation, without involving a preferred foliation of spacetime.

For simplicity, in this paper we study only a real scalar field. However, it is straightforward to apply the developed MFT formalism to a Bohmian interpretation of any other quantum field, including fermionic fields [22] and the gravitational field [3, 6, 7, 8, 9].

In the paper we use units $\hbar = c = 1$, while the signature of spacetime metric is $(+,-,-,-)$.

2 MFT formulation of QFT

Let $x = \{x^\mu\} = (x^0, \mathbf{x})$ be spacetime coordinates. A timelike Cauchy hypersurface Σ can be defined by a function $T(\mathbf{x})$, through the equation

$$x^0 = T(\mathbf{x}). \tag{1}$$

The coordinates $\mathbf{x}$ may be used as coordinates on Σ. If $T(\mathbf{x})$ is given for all $\mathbf{x}$, then $\mathbf{x}$ can also be viewed as a point on Σ and allows us to write $\mathbf{x} \in \Sigma$. Let $\phi(\mathbf{x})$ be a dynamical field on Σ. (For simplicity, we take it to be a real scalar field.) To avoid notational confusion, from now on $\phi(\mathbf{x})$ and $T(\mathbf{x})$ denote a value at a point $\mathbf{x}$, while ϕ and T without an argument denote a set of values at *all* points $\mathbf{x}$. Similarly, if σ is a subset of Σ, then $\phi|_\sigma$ and $T|_\sigma$ denote a set of values at all points of σ. (With this notation, $\phi = \phi|_\Sigma$, $T = T|_\Sigma$.) Let $\hat{\mathcal{H}}(\mathbf{x})$ be the Hamiltonian-density operator. The dynamics of the field ϕ is described by the MFT Tomonaga-Schwinger equation

$$\hat{\mathcal{H}}(\mathbf{x})\Psi[\phi, T] = i\frac{\delta\Psi[\phi, T]}{\delta T(\mathbf{x})}. \tag{2}$$

The wave functional $\Psi[\phi, T]$ can also be viewed as a functional of $\phi|_\Sigma$, where Σ is defined by T. Eq. (2) describes how Ψ changes for an infinitesimal change $\delta T(\mathbf{x})$ of the hypersurface Σ. One easily recovers the usual functional Schrödinger equation by integrating (2) over the entire hypersurface Σ, provided that one considers a special form of the variation $\delta T(\mathbf{x})$, such that $\delta T(\mathbf{x})$ does not depend on $\mathbf{x}$. Thus we see that (2) represents a generalization of the ordinary Schrödinger equation. However, in contrast to the ordinary Schrödinger equation, the right-hand side of (2) does not involve any preferred foliation of spacetime. We also note that, in the original papers [16, 17], $\hat{\mathcal{H}}(\mathbf{x})$ was the interaction Hamiltonian density in the interaction picture, while here, in accordance with [18, 19], $\hat{\mathcal{H}}(\mathbf{x})$ is the total Hamiltonian density in the Schrödinger picture.

The quantity $\rho[\phi, T] = |\Psi[\phi, T]|^2$ represents the probability density for the field to have a value ϕ on Σ. Since Σ is determined by T, it is also convenient to say that $\rho[\phi, T]$ is the probability density for the field to have a value ϕ at time T. However, when using the latter terminology, it is important to remember that T is not a single real parameter, but a collection of an infinite number of real parameters, with one real parameter for each point $\mathbf{x}$.

Now consider a quantum measurement. It is convenient to describe a measurement in terms of a wave-functional collapse. The collapse can be described in a general way as follows. A normalized solution $\Psi[\phi, T]$ of (2) can be written as a linear combination of other orthonormal solutions as

$$\Psi[\phi, T] = \sum_a c_a \Psi_a[\phi, T]. \tag{3}$$

For any given Ψ_a and any given T_0, there exists a hermitian operator such that $\Psi_a[\phi, T_0]$ is an eigenvector of this operator. Since any hermitian operator corresponds to a quantity that can, at least in principle, be measured, a measurement at T_0 may induce a wave-functional collapse of the form

$$\Psi[\phi, T] \to \Psi_a[\phi, T]. \tag{4}$$

The probability for the collapse (4) is $|c_a|^2$. In particular, $\Psi_a[\phi, T]$ may be a product of the form

$$\Psi_a[\phi, T] = A[\phi|_{\bar{\sigma}}, T|_{\bar{\sigma}}] \prod_{\mathbf{x} \in \sigma} \delta(\varphi(\mathbf{x}) - \phi(\mathbf{x})), \tag{5}$$

where σ is a subset of Σ and $\bar{\sigma} \equiv \Sigma - \sigma$ denotes the set of all points of Σ that are not contained in σ. The state (5) has a well defined value of $\phi|_\sigma$. Therefore, if (5) is satisfied, then the collapse (4) may occur by measuring $\phi|_\sigma$ at $T_0|_\sigma$, without any measurement on $\bar{\sigma}$. (This is similar to the fact that the total wave function of an entangled Einstein-Podolsky-Rosen (EPR) pair collapses by measuring the spin of one member of the pair, without measuring the spin of the other member. For a MFT description of the EPR effect, see [23].) In this sense, a measurement can assign a definite value of $\phi|_\sigma$ on σ, without saying anything about $T|_{\bar{\sigma}}$ and $\phi|_{\bar{\sigma}}$. In particular, this implies that, in the MFT formulation, the wave-functional collapse does not need to be "instantaneous", because the time of measurement $T_0|_\sigma$ does not determine the "collapse time" $T_0|_{\bar{\sigma}}$ of the unmeasured variables.

A measurement can also be described more accurately by introducing the degrees of freedom χ of the measuring apparatus. If the apparatus measures the eigenvalues associated with the states Ψ_a, then the total wave functional describing the entanglement between the measured system and the measuring apparatus takes the form

$$\Psi[\phi, \chi, T] = \sum_a c_a \Psi_a[\phi, T] \Theta_a[\chi, T]. \tag{6}$$

Here $\Theta_a[\chi, T]$ are orthonormal functionals that do not overlap, in the sense that

$$\Theta_a[\chi, T] \Theta_{a'}[\chi, T] = 0 \quad \text{for} \quad a \neq a'. \tag{7}$$

Therefore, if χ at T_0 is found to have a value that belongs to the support of Θ_a, then the value of χ at T_0 does not belong to the support of any other $\Theta_{a'}$. In this case, the total effective wave functional is given by $\Psi_a[\phi, T]\Theta_a[\chi, T]$. Consequently, the effective wave functional describing only the ϕ-degree od freedom is $\Psi_a[\phi, T]$, which corresponds to the collapse (4). The probability for this effective collapse is equal to $|c_a|^2$. Of course, the conventional interpretation of quantum mechanics does not say what, if anything, causes χ to take a definite value. However, such a cause is provided by the Bohmian interpretation discussed in the next section.

3 Bohmian interpretation

For simplicity, consider a free scalar field with the Hamiltonian density

$$\hat{\mathcal{H}}(\mathbf{x}) = -\frac{1}{2} \frac{\delta^2}{\delta \phi^2(\mathbf{x})} + \frac{1}{2}[(\nabla \phi(\mathbf{x}))^2 + m^2 \phi^2(\mathbf{x})]. \tag{8}$$

By writing $\Psi = Re^{iS}$, where R and S are real functionals, the complex equation (2) is equivalent to a set of two real equations

$$\frac{1}{2}\left(\frac{\delta S}{\delta \phi(\mathbf{x})}\right)^2 + \frac{1}{2}[(\nabla \phi(\mathbf{x}))^2 + m^2 \phi^2(\mathbf{x})] + \mathcal{Q}(\mathbf{x}; \phi, T] + \frac{\delta S}{\delta T(\mathbf{x})} = 0, \tag{9}$$

$$\frac{\delta\rho}{\delta T(\mathbf{x})} + \frac{\delta}{\delta\phi(\mathbf{x})}\left(\rho\frac{\delta S}{\delta\phi(\mathbf{x})}\right) = 0, \tag{10}$$

where $\rho = R^2$ and

$$\mathcal{Q}(\mathbf{x};\phi,T] = -\frac{1}{2R}\frac{\delta^2 R}{\delta\phi^2(\mathbf{x})}. \tag{11}$$

The conservation equation (10) shows that it is consistent to interpret $\rho[\phi,T]$ as the probability density for the field to have the value ϕ at the hypersurface determined by T.

The Bohmian interpretation consists in introducing a deterministic time-dependent hidden variable, such that the time evolution of this variable is consistent with the probabilistic interpretation of ρ. From (10), we see that this is naturally achieved by introducing a MFT field $\Phi(\mathbf{x};T]$ that satisfies the MFT Bohmian equation of motion

$$\frac{\delta\Phi(\mathbf{x};T]}{\delta T(\mathbf{x}')} = \delta^3(\mathbf{x}-\mathbf{x}')\left.\frac{\delta S}{\delta\phi(\mathbf{x})}\right|_{\phi=\Phi}. \tag{12}$$

It can also be integrated over d^3x' inside an arbitrarily small region $\sigma_{\mathbf{x}}$ around $\mathbf{x}$, to yield

$$\int_{\sigma_{\mathbf{x}}} d^3x' \frac{\delta\Phi(\mathbf{x};T]}{\delta T(\mathbf{x}')} = \left.\frac{\delta S}{\delta\phi(\mathbf{x})}\right|_{\phi=\Phi}. \tag{13}$$

Eq. (13) is the MFT version of the usual single-time Bohmian equation of motion $\partial\Phi(\mathbf{x},t)/\partial t = \delta S/\delta\phi(\mathbf{x})|_{\phi=\Phi}$. However, (12) is more fundamental than (13) because (12) does not involve an arbitrary region $\sigma_{\mathbf{x}}$. On the other hand, the integration inside $\sigma_{\mathbf{x}}$ is useful for an easier comparison with the conventional single-time formalism. For example, from (12) and the quantum MFT Hamilton-Jacobi equation (9), one finds that $\Phi(\mathbf{x};T]$ satisfies

$$\left[\left(\int_{\sigma_{\mathbf{x}}} d^3x' \frac{\delta}{\delta T(\mathbf{x}')}\right)^2 - \nabla_{\mathbf{x}}^2 + m^2\right]\Phi(\mathbf{x};T] = -\int_{\sigma_{\mathbf{x}}} d^3x' \left.\frac{\delta\mathcal{Q}(\mathbf{x}';\phi,T]}{\delta\phi(\mathbf{x})}\right|_{\phi=\Phi}. \tag{14}$$

This can be viewed as a MFT Klein-Gordon equation, modified with a nonlocal quantum term on the right-hand side.

The δ-function on the right-hand side of (12) implies that the left-hand side of (12) vanishes when $\mathbf{x}' \neq \mathbf{x}$. This implies that $\Phi(\mathbf{x};T]$ does not depend on the whole function T, but only on $T(\mathbf{x})$. Therefore, we have

$$\Phi(\mathbf{x};T] = \Phi(\mathbf{x},T(\mathbf{x})). \tag{15}$$

Using (1), we can also write

$$\Phi(\mathbf{x},T(\mathbf{x})) = \Phi(\mathbf{x},x^0) = \Phi(x). \tag{16}$$

Eq. (15) shows that Φ is a MFT object depending on many times, with one time $T(\mathbf{x})$ for each $\mathbf{x}$. Owing to this MFT property, one does not need a preferred foliation of spacetime. On the other hand, Eq. (16) shows that, at the kinematic level, Φ can also be viewed as a usual local field in which the MFT nature is not manifest.

Let us also say a few words on quantum measurements. The basics of MFT quantum measurements are described in Sec. 2, while more details on quantum measurements in the Bohmian context can be found in [1, 2, 4, 5]. Here it suffices to say that the χ-degree of freedom of the measuring apparatus takes a definite value because χ is also a deterministic degree of freedom obeying a Bohmian equation of motion. Consequently, owing to (7), the Bohmian equation of motion for ϕ takes a form that it would take if $\Psi[\phi,T]$ were equal to one of Ψ_a's in (6). This is how the Bohmian interpretation explains the effective collapse of the wave functional.

Now let us give a few notes on the classical limit. The classical Hamilton-Jacobi equation can also be formulated as a MFT theory [24, 25]. The equation of motion can also be written in the MFT form (12). However, in the classical case, the nonlocal $\mathcal{Q}$-term in (9) does not appear, so that the dynamics is completely local.

4 Manifestly covariant formulation

The MFT formalism was introduced by Tomonaga and Schwinger with the intention to provide the manifest covariance of QFT in the interaction picture. On the other hand, in this paper we are using the Schrödinger picture, and our presentation in the preceding sections is not manifestly covariant. Although the MFT formalism automatically avoids the introduction of a preferred foliation of spacetime, in the preceding sections time is not treated on an equal footing with space. Fortunately, the MFT formalism can be formulated in a manifestly covariant way [18] (see also [24, 25]). In this section we briefly review the basics of this covariant formalism and apply the formalism to the Bohmian interpretation.

We start by introducing a set of 3 real parameters $\{s^1, s^2, s^3\} \equiv \mathbf{s}$ that serve as coordinates on a 3-dimensional manifold. *A priori*, there is not any relation between these coordinates and the spacetime coordinates x^μ. However, the 3-dimensional manifold can be embedded in the 4-dimensional spacetime by introducing 4 functions $X^\mu(\mathbf{s})$. A 3-dimensional hypersurface in spacetime is defined by the set of 4 equations

$$x^\mu = X^\mu(\mathbf{s}). \tag{17}$$

The 3 parameters s^i can be eliminated, leading to one equation of the form $f(x^0, x^1, x^2, x^3) = 0$, which, indeed, is an equation that determines a 3-dimensional hypersurface in spacetime. Assuming that the background spacetime metric $g_{\mu\nu}(x)$ is given, the induced metric $q_{ij}(\mathbf{s})$ on the hypersurface is

$$q_{ij}(\mathbf{s}) = g_{\mu\nu}(X(\mathbf{s}))\frac{\partial X^\mu(\mathbf{s})}{\partial s^i}\frac{\partial X^\nu(\mathbf{s})}{\partial s^j}, \tag{18}$$

where $X \equiv \{X^\mu\}$. Similarly, a normal to the surface is

$$\tilde{n}_\mu(\mathbf{s}) = \epsilon_{\mu\alpha\beta\gamma}\frac{\partial X^\alpha}{\partial s^1}\frac{\partial X^\beta}{\partial s^2}\frac{\partial X^\gamma}{\partial s^3}. \tag{19}$$

The unit normal transforming as a spacetime vector is

$$n^\mu(\mathbf{s}) = \frac{g^{\mu\nu}\tilde{n}_\nu}{\sqrt{|g^{\alpha\beta}\tilde{n}_\alpha\tilde{n}_\beta|}}. \tag{20}$$

Now equations of the preceding sections can be written in a covariant form by making the replacements

$$\mathbf{x} \to \mathbf{s}, \qquad \frac{\delta}{\delta T(\mathbf{x})} \to \frac{\delta}{\delta\tau(\mathbf{s})} \equiv n^\mu(\mathbf{s})\frac{\delta}{\delta X^\mu(\mathbf{s})}. \tag{21}$$

The Tomonaga-Schwinger equation (2) becomes

$$\hat{\mathcal{H}}(\mathbf{s})\Psi[\phi, X] = in^\mu(\mathbf{s})\frac{\delta\Psi[\phi, X]}{\delta X^\mu(\mathbf{s})}. \tag{22}$$

For free fields, the Hamiltonian-density operator in curved spacetime is

$$\hat{\mathcal{H}} = \frac{-1}{2|q|^{1/2}}\frac{\delta^2}{\delta\phi^2(\mathbf{s})} + \frac{|q|^{1/2}}{2}[-q^{ij}(\partial_i\phi)(\partial_j\phi) + m^2\phi^2], \tag{23}$$

where q is the determinant of q_{ij}. The Bohmian equation of motion (12) becomes

$$\frac{\delta\Phi(\mathbf{s};X]}{\delta\tau(\mathbf{s}')} = \frac{\delta^3(\mathbf{s}-\mathbf{s}')}{|q(\mathbf{s})|^{1/2}} \left.\frac{\delta S}{\delta\phi(\mathbf{s})}\right|_{\phi=\Phi}. \tag{24}$$

Similarly, (14) becomes

$$\left[\left(\int_{\sigma_{\mathbf{s}}} d^3s' \frac{\delta}{\delta\tau(\mathbf{s}')}\right)^2 + \nabla^i\nabla_i + m^2\right]\Phi(\mathbf{s};X] = -\int_{\sigma_{\mathbf{s}}} \frac{d^3s'}{|q(\mathbf{s})|^{1/2}} \left.\frac{\delta\mathcal{Q}(\mathbf{s}';\phi,X]}{\delta\phi(\mathbf{s})}\right|_{\phi=\Phi}, \tag{25}$$

where ∇_i is the covariant derivative with respect to s^i and

$$\mathcal{Q}(\mathbf{s};\phi,X] = -\frac{1}{|q(\mathbf{s})|^{1/2}} \frac{1}{2R} \frac{\delta^2 R}{\delta\phi^2(\mathbf{s})}. \tag{26}$$

The same hypersurface Σ can be parametrized by different sets of 4 functions $X^\mu(\mathbf{s})$. On the other hand, the quantities such as $\Psi[\phi,X]$ and $\Phi(\mathbf{s};X]$ depend on Σ, but do not depend on the way in which Σ is parametrized. The freedom in choosing functions $X^\mu(\mathbf{s})$ is a sort of gauge freedom closely related to the covariance. To find a solution of the covariant equations above, it is convenient to fix a gauge. For a timelike hypersurface, the simplest choice of gauge is

$$X^i(\mathbf{s}) = s^i. \tag{27}$$

This choice implies $\delta X^i(\mathbf{s}) = 0$, which leads to equations similar to those of Secs. 2 and 3. For example, (24) becomes

$$(g^{00}(\mathbf{x}))^{1/2} \frac{\delta\Phi(\mathbf{x};X^0]}{\delta X^0(\mathbf{x}')} = \frac{\delta^3(\mathbf{x}-\mathbf{x}')}{|q(\mathbf{x})|^{1/2}} \left.\frac{\delta S}{\delta\phi(\mathbf{x})}\right|_{\phi=\Phi}, \tag{28}$$

which is the curved-spacetime version of (12).

The MFT formalism can also be formulated in a manifestly covariant way by introducing a more abstract formalism [17] that does not involve parametrizations such as (1) or (17). A functional F of the hypersurface is written as $F[\Sigma]$, while the derivative operator in (21) is written as $\delta/\delta\Sigma(x)$. This derivative is defined as

$$\frac{\delta F[\Sigma]}{\delta\Sigma(x)} = \lim_{v_x^{(4)}\to 0} \frac{F[\Sigma'] - F[\Sigma]}{v_x^{(4)}}, \tag{29}$$

where $v_x^{(4)}$ is the 4-volume around x enclosed between the hypersurfaces Σ' and Σ. In this language, the Bohmian equation of motion (24) can be written in another covariant form as

$$\frac{\delta\Phi(x;\Sigma]}{\delta\Sigma(x')} = \frac{\delta^3_\Sigma(x-x')}{|q_\Sigma(x)|^{1/2}} \left.\frac{\delta S[\phi,\Sigma]}{\delta_\Sigma\phi(x)}\right|_{\phi=\Phi}, \tag{30}$$

and similarly for other equations. Here the subscript Σ denotes quantities attributed to the hypersurface Σ.

5 Conclusion

The MFT formulation of QFT does not involve a preferred foliation of spacetime, which allows a covariant formulation of QFT. This naturally leads to a covariant MFT Bohmian interpretation of quantum fields, which also does not involve a preferred foliation of spacetime. The covariant dynamics of Bohmian fields does not depend on the choice of coordinates. When a particular set of coordinates is chosen, then the *solution* of the MFT Bohmian equation of motion can be written such that the MFT nature of the field is not manifest. However, the Bohmian *equation of motion* itself retains its manifest MFT form.

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